Module 9

Pedagogy of Mathematics

Overview

Mathematics finds an important place in school education. At elementary stage, it is a compulsory subject. However, many people opine that it creates fear, phobia and stress on children. Classroom observations often point towards, way of teaching of this subject create this stress. In view of addressing this concern, this module emphasises upon developing competencies and skills in mathematics at the primary and upper primary stage children through learner-friendly pedagogy that integrates assessment and also engage all children with different abilities in a stress free classroom environment.

Learning Objectives

- After going through this module the teachers will know about the transactional strategies including the assessment part that can be adopted to engage the children in learning. They will be able to
  - relate the competencies and skills as given in the Learning outcomes with the state syllabus
  - conduct appropriate pedagogical processes to help children in achieving the class level learning outcomes
  - integrate assessment with pedagogical processes to continuously ensure the progress in learning by all children

Understanding the Nature of Mathematics

Mathematics has helped us quantify ideas, to be precise and to utilise spatial concepts in our day-to-day living. It is used throughout the world as an essential tool in many fields including natural science, engineering, medicine and social science. Mathematics not only helps in day-to-day situations but also develops logical reasoning, abstract thinking and imagination. Thus, it has occupied an important place in the school curriculum and is a compulsory subject upto Class X.

Some problems in school Mathematics education

- Child—When I multiply two natural numbers the product is bigger than both the numbers but when I multiply two fractions the product is smaller. I am not able to understand why.

- A majority of children have a sense of fear and failure regarding Mathematics. Hence, they give up early on, and drop out of serious mathematical learning.
- The curriculum is disappointing not only to this non participating majority, but also to the talented minority by offering them no challenges.
- Problems, exercises and methods of evaluation are mechanical and repetitive, with too much emphasis on computation. Areas of Mathematics, such as spatial thinking are not developed enough in the curriculum.
- Teachers lack confidence, preparation and support.

(NCF-2005)

Developing Numeracy skills among children at the pre-primary (Pre-school) stage

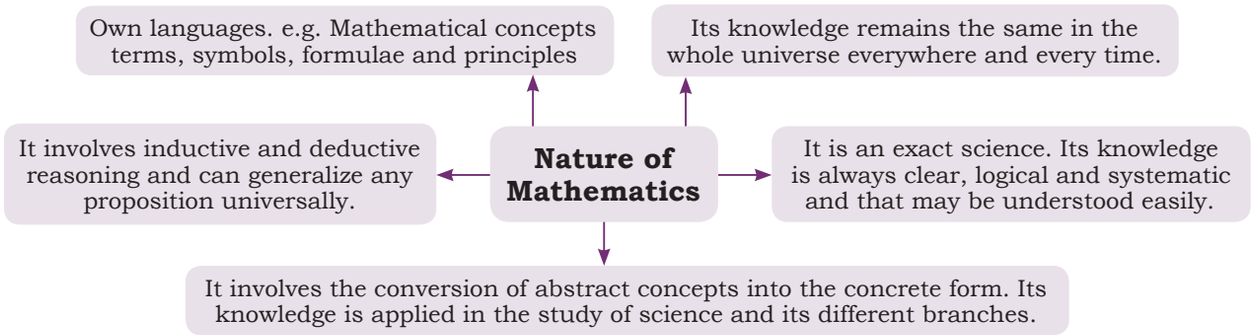
- Pre-numbers Skills—Classification, sorting, ordering and one-to-one correspondences are the skills which form the basis for development of numeracy skills and abilities.
- The children are expected to be competent in the pre-number skills before learning to count the objects. A child should be given ample time and opportunities to strengthen the pre-number skills. Practical activities related to classification of objects (on the basis of one or more attributes), ordering of concrete objects (in different ways) and one-to-one correspondence should be done with the child as these skills form a base for a child who is learning to count.
- At pre-school stage you can help your child begin to develop early math skills and abilities by introducing ideas like—Number sense (the ability to count accurately forward and backward, and to see relationships between numbers through basic operations); Representation (using words, pictures, symbols, and objects, like blocks, pebbles etc.); Spatial sense (the idea of shape, size, space, position, direction, and movement); Measurement (finding the length, height, weight and time using their own ways and later by using units); Estimation (ability to make a good guess about the amount or size of something); Patterns (arrangements of numbers, shapes, images—that repeat in a logical way) and Problem-solving(the ability to think through a problem and to recognise there is more than one path to the answer).
- In order to help children to develop above abilities it is essential that the experiences that children bring in from home to be used

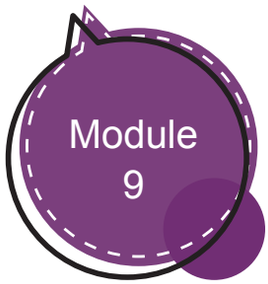
optimally instead of creating artificial situations. For example—for the ability of classification, children should be provided

- opportunities to classify objects, people, material etc. in their close vicinity firstly in two groups with clearly observable attributes like color and shape, and later in more than two groups and with slightly complex properties like size, position, orientation etc.

This leads us to an important question as to, what is the main goal of mathematics education at school level. Do we merely want to develop the numeracy related skills in the students and make them employable adults or do we want to make them thinking individuals who in a longer run would contribute to social and economic development of the society? It is for this concern of developing thinking individuals in the society that mathematics education at school level assumes greater importance.

In this module we are going to discuss the ways in which we can make the students think about mathematics and mathematise their thought processes. Let us begin by looking at what is mathematics. The word mathematics has been derived from the Greek word 'Mathema' which means science. *The Oxford Dictionary* defines mathematics as "The science of space, numbers and quantity". Primarily mathematics is a method of inquiry study of measurements, patterns and symmetry. The method contains a careful formulation of definitions of the concepts to be discussed and an explicit statement of the assumptions that form the basis of reasoning. From these definitions and assumptions conclusions are deduced by application of rigorous logic.





Mathematics at the Primary Level

As a society we are yet to appreciate how crucial the work of the primary school teacher is, for society as a whole. The primary teacher carries a large responsibility and is the one who can make a difference to how and what the child learns in the later years of school. Teaching at the primary level requires a specific kind of expertise which includes both knowledge and skills. Such expertise is gained over a period of time, and gained through thinking about one's work and making a continuous effort to improve one's teaching. Children's thinking at this stage majorly spirals around concrete objects that too in their familiar contexts. Thus, all the teaching-learning processes should focus on engaging the children in manipulation of concrete objects. The pedagogy at this stage includes the following sequence of activities while dealing with different concepts, competencies and skill—

Vision for School Mathematics

- Children learn to enjoy mathematics rather than fear it.
- Children learn important mathematics—Mathematics is more than formula and mechanical procedures.
- Children see mathematics as something to talk about, to communicate through, to discuss among themselves, to work together on.
- Children pose and solve meaningful problems.
- Children use abstractions to perceive relation-ships, to see structures, to reason out things, to argue the truth or falsity of statements.
- Children understand the basic structure of Mathematics—Arithmetic, algebra, geometry and trigonometry, the basic content areas of school Mathematics, all offer a methodology for abstraction, structuration and generalisation.
- Teachers engage every child in class with the conviction that everyone can learn mathematics

(NCF-2005)

Experiences

Providing lot of experiences with concrete objects and manipulatives like toys, learning aids, etc. is the key to initiate any new concept. The experiences at this stage should have strong linkage with the activities inside and outside the class rooms. The assignments to be given to the children should

be of exploratory manner so that they can relate the learning with their day-to-day life. For example, for addition of numbers let children have two groups of objects from their vicinity and recount the objects after combining the two groups. Initially the groups of same type of objects should be provided like apples or any fruit so that they can count after combining and telling the total number of apples. After the children gain confidence in combining such groups and find the sum of numbers, provide them the opportunities for problem posing. If you know that 235 + 367 = 602, how much is 234 + 369? How did you find the answer? Change any one digit in 5384. Did the number increase or decrease? Now let the child combine different groups of objects with some common property, like one group of boys and one of girls when combined may form a group of students/children. Initially the sum of the numbers is obtained by recounting all objects in the combined group. Later the children may devise their ways of counting ahead in place of recounting all the objects.

Language

Provide appropriate opportunities to all children to verbally describe their experiences, observations and hypothesis. During such discussion children will develop language skills also like framing questions, acquiring newer vocabulary and terminology related to the subject. For example, when a group of five stars is combined with another group of three stars the total number of stars in the finally obtained group is eight.

Pictorial presentation

Involve children interpreting pictures and to find relevant information. Such pictures may even show a problem and its solutions. For example,

In higher classes this will facilitate the children to learn to represent the mathematical ideas/solutions diagrammatically/ pictorially.

Symbols

In the and the children should be provided opportunities to represent and interpret information using symbols.

For example,

5 + 3 = 8

Encourage learners to represent their observations as symbols and attach meaning to them. Also encourage them to relate symbols with signs of fundamental operations to make them mathematical expression or equation.

This will help children in presenting their ideas in a precise and lucid manner. The development of other competencies related to reading and writing, expression of ideas and problem solving will take place in an integrated manner.

Integration of language and environmental studies with mathematics

In early school classes, i.e., Class I and II, children study only one or two languages and mathematics. But the requirements of understanding their surroundings, language as a tool for better communication and understanding and using mathematical terms and ideas in problem solving are integrated in these subjects only. Especially when teaching-learning of early mathematics is taking place the teachers should focus on all round development of the child along with outcomes related to mathematics, languages and EVS which are developed in an integrated manner through examples from child's context and surroundings. For example while talking about numbers, the discussion about where these numbers occur in their surrounding; how they use numbers and number operations in their daily life; how can they describe different situations without using number names and symbols;, etc., should be held in the classrooms. With such discussion they must realise that mathematical terminology and symbols are strong parts of any language. For example while discussing about the measurements the units used are developed through the daily usage and form the basis for communication in any language.

The division of knowledge into various subjects has been done in the syllabus. For the child schooling is the process to find answers to many questions that arise in his/her mind while interacting with the surrounding. Let children enjoy this holistic quest within and outside schools and beyond the boundaries of subjects.

Mathematics at the Upper Primary Stage

Mathematics at the upper primary stage is a major challenge and has to perform the dual role of being both close to the experience and environment of the child and being abstract. Children often are not able to work in terms of ideas alone. They need the comfort of context and/or models linked to their

experience to find meaning. This stage presents before us the challenge of engaging the children while using the contexts but gradually moving them away from such dependence. So while children should be able to identify the principles to be used in a contextual situation, they should not be dependent or be limited to contexts. As we progress further in the middle school there would be greater requirement from the child to be able to do this. For example, upto class V children visualised numbers through which counting can be done (natural numbers) and in the Class VI the idea of negative numbers and integers comes to child's discussion. Different patterns in numbers have been observed by the child and may have generalised also. At upper primary stage children start learning algebra as generalisation of arithmetic. This helps in putting the generalisation in precise manner by writing symbols/letters for numbers and also in establishing proofs of the generalisations that have been made after observation of number patterns. Moreover, the study of geometry at this stage aims at analysing various 2-D and 3-D shapes to explore and generalise their properties based on edges, vertices, angles and surfaces. Children should be competent to make rules on generalisation of properties like sum of interior angles of a rectilinear shape (polygon) is 2 less than number of sides times 180 degrees, a triangle and its congruent triangle together form a parallelogram, therefore measure of area of a triangle is half the measure of the area of the parallelogram. Later the child is expected to develop proofs for such generalisations which continues till class X and XII.

Pedagogical Processes

A number of factors may influence the learning of mathematics but teachers play an important role in the performance in mathematics. It is imperative, therefore, that we understand what effective mathematics teaching looks like—and what can teachers do to break this pattern. The common belief in society is that if a mathematics teacher knows mathematics very well, he or she is the best person to teach mathematics. But what about "knowing to teach mathematics"? The knowledge in mathematics alone won't help a person to teach mathematics. He/she should also have sound knowledge in the area of how to teach mathematics. The knowledge in mathematics and how to teach mathematics together is commonly known as Pedagogical Content Knowledge (PCK). Following are some key actions required for making mathematics joyful—

- Participation

- Engagement
- Observations
- Making hypothesis and verifying them
- Problem posing
- Problem solving
- Visualisation and representation
- Making connections
- Systematic reasoning
- Mathematical communication

In National Achievement Survey 2017 which was based on Learning Outcomes, percentage of correct responses (on an Average) for class III, V and VIII in Mathematics at the National level has found as follows.
Class III — 64%
Class V — 53%
Class VIII — 42%
Do we know about state/UT average achievement and district average achievement? Details are available on http://www.ncert.nic.in/programmes/NAS/SRC.html. We need to reflect on how to improve learning outcomes of our students in Mathematics.

Class-wise Learning outcomes

Class I

The learner

- classifies objects into groups based on a few physical attributes, such as shape, size and other observable properties including rolling and sliding recites number names and counts objects up to 20, concretely, pictorially and symbolically.
- works with numbers 1 to 20.
  - counts objects using numbers 1 to 9.
  - compares numbers up to 20. For example, tell whether number of girls or number of boys is more in the class.
- applies addition and subtraction of numbers 1 to 20 in daily life.
  - constructs addition facts up to 9 by using concrete objects. For example to find 3+3 counts 3 steps forward from 3 and concludes that 3+3=6.

- subtracts numbers using 1 to 9. For example the child takes out 3 objects from a collection of 9 objects and counts the remaining to conclude 9–3=6.
- solves day-to-day problems related to addition and subtraction of numbers up to 9.

- recognises numbers up to 99 and writes numerals.
- describes the physical features of various solids/shapes in her own language. For example, a ball rolls, a box slides etc.
- estimates and measures short lengths using non-uniform units like a finger, hand span, length of a forearm, footsteps, etc.
- observes, extends and creates patterns of shapes and numbers. For example, arrangement of shapes/objects/ numbers, etc.
  - 1,2,3,4,5,...
  - 1,3,5,...
  - 2,4,6,...
  - 1,2,3,1,2,..., 1,...3,...
- collects, records (using pictures/numerals) and interprets simple information by looking at visuals. (For example in a picture of a garden the child looks at different flowers and draws inference that flowers of a certain colour are more).
- develops the concept of zero.

Class II

The learner

- works with two digit numbers.
  - reads and writes numerals for numbers up to 99.
  - uses place value in writing and comparing two digit numbers.
  - forms the greatest and smallest two digit numbers (with and without repetition of given digits).
  - solves simple daily life problems/situations based on addition of two digit numbers.
  - solves daily life situations based on subtraction of two digit numbers.
  - represents an amount up to Rs.100 using 3–4 notes and coins (of same/different denominations of play money.
- describes basic 3D and 2D shapes with their observable characteristics.
  - identifies basic 3D-shapes such as cuboid, cylinder, cone and sphere by their names.

- distinguishes between straight and curved lines.
- draws/represents straight lines in various orientations (vertical, horizontal, slant).

- estimates and measures length/distances and capacities of containers using uniform non-standard units like a rod/ pencil, cup/spoon/bucket etc.
- compares objects as heavier/lighter using simple balance.
- identifies the days of the week and months of the year.
- sequences the events occurring according to their duration in terms of hours/days; for example, Does a child remain in school for a longer period than at home?
- draws inference based on the data collected such as the number of vehicles used in Samir's house is more than that in Angelina's.

Class III

The learner

- works with three digit numbers
  - reads and writes numbers up to 999 using place value.
  - compares numbers up to 999 for their value based on their place value.
  - solves simple daily life problems using addition and subtraction of three digit numbers with and without regrouping, sums not exceeding 999.
  - constructs and uses the multiplication facts (tables) of 2, 3, 4, 5 and 10 in daily life situations.
  - analyses and applies an appropriate number operation in the situation/context.
  - explains the meaning of division facts by equal grouping/ sharing and finds it by repeated subtraction. For example, $12 \div 3$ can be explained as number of groups of 3 to make 12 and finds it as 4 by repeatedly subtracting 3 from 12.
- adds and subtracts small amounts of money with or without regrouping.
- makes rate charts and simple bills.
- acquires understanding about 2D shapes.
  - identifies and makes 2D-shapes by paper folding , paper cutting on the dot grid, using straight lines etc.
  - describes 2D shapes by the number of sides, corners and diagonals. For example, the shape of the book cover has 4 sides, 4 corners and two diagonals.

- fills a given region leaving no gaps using a tile of a given shape.

- estimates and measures length and distance using standard units like centimetres or metres and identifies relationships.
- weighs objects using standard units – grams and kilograms using simple balance.
- compares the capacity of different containers in terms of non standard units.
- adds and subtracts measures involving grams & kilograms in life situations.
- identifies a particular day and date on a Calendar.
- reads the time correctly to the hour using a clock/watch.
- extends patterns in simple shapes and numbers.
- records data using tally marks, represents pictorially and draws conclusions.

Class IV

The learner

- applies operations of numbers in daily life.
  - multiplies 2 and 3 digit numbers.
  - divides a number by another number using different methods like—pictorially (by drawing dots), equal grouping or repeated subtraction and by using inter-relationship between division and multiplication.
  - creates and solves simple real life situations/problems including money, length, mass and capacity by using the four operations.
- works with fractions.
  - identifies half, one-fourth, three-fourths of a whole in a given picture by paper folding and also in a collection of objects.
  - represents the fractions as half, one-fourth and three-fourths by using numbers/numerals.
  - shows the equivalence of a fraction with other fractions.
- acquires understanding about shapes around her/him.
  - identifies the centre, radius and diameter of the circle.
  - finds out shapes that can be used for tiling.
  - makes cube/cuboids using the given nets.
  - shows through paper folding/paper cutting, ink blots, etc. the concept of symmetry by reflection.
  - draws top view, front view and side view of simple objects.

- explores the area and perimeter of simple geometrical shapes (triangle, rectangle.
- square) in terms of given shape as a unit. For example, the number of books that can completely fill the top of a table.
- converts metre into centimetre and vice-versa.
- estimates the length of an object/distance between two locations, weight of various objects, volume of liquid, etc., and verifies them by actual measurement.
- solves problem involving daily life situations related to length, distance, weight, volume and time involving four basic arithmetic operations.
- reads clock time in hour and minutes and expresses the time in a.m. and p.m.
- relates to 24-hr-clock with respect to 12 hr-clock.
- calculates time intervals/duration of familiar daily life events by using forward or backward counting/addition and subtraction.
- identifies the pattern in multiplication and division (up to multiple of 9).
- observes, identifies and extends geometrical patterns based on symmetry.
- represents the collected information in tables and bar graphs and draws inferences from these.

Class V

The learner

- works with large numbers.
  - reads and writes numbers bigger than 1000 being used in her/his surroundings.
  - performs four basic arithmetic operations on numbers beyond 1000 by understanding of place value of numbers.
  - divides a given number by another number using standard algorithms.
  - estimates sum, difference, product and quotient of numbers and verifies the same using different strategies like using standard algorithms or breaking a number and then using operation. For example, to divide 9450 by 25, divide 9000 by 25, 400 by 25, and finally 50by 25 and gets the answer by adding all these quotients.
- acquires understanding about fractions.
  - finds the number corresponding to part of a collection
  - identifies and forms equivalent fractions of a given fraction

  - expresses a given fraction 1/2, 1/4, 1/5 in decimal notation and vice-versa. For example, in using units of length and money– half of Rs.10 is Rs.5.
  - converts fractions into decimals and vice versa.
- explores idea of angles and shapes.
  - classifies angles into right angle, acute angle, obtuse angle and represents the same by drawing and tracing.
  - identifies 2D shapes from the immediate environment that have rotation and reflection symmetry like alphabet and shapes.
  - makes cube, cylinder and cone using nets designed for this purpose.
- relates different commonly used larger and smaller units of length, weight and volume and converts larger units to smaller units and vice-versa.
- estimates the volume of a solid body in known units like volume of a bucket is about 20 times that of a mug.
- applies the four fundamental arithmetic operations in solving problems involving money, length, mass, capacity and time intervals.
- identifies the pattern in triangular number and square number.
- collects data related to various daily life situations, represents it in tabular form and as bar graphs and interprets it.

Class VI

The learner

- solves problems involving large numbers by applying appropriate operations (addition, subtraction, multiplication and division).
- recognises and appreciates (through patterns) the broad classification of numbers as even, odd, prime, co-prime, etc.
- applies HCF or LCM in a particular situation.
- solves problem involving addition and subtraction of integers.
- uses fractions and decimals in different situations which involve money, length, temperature, etc. For example, 7½ metres of cloth. distance between two places is 112.5 km etc.
- solves problems on daily life situations involving addition and subtraction of fractions/decimals.
- uses variable with different operations to generalise a given situation. for example, Perimeter of a rectangle with sides x units and 3 units is 2(x+3) units.

- compares quantities using ratios in different situations. for example, the ratio of girls to boys in a particular class in 3:2.
- uses unitary method in solving various word problems. For example, if the cost of a dozen notebooks is given she finds the cost of seven notebooks by first finding the cost of 1 notebook.
- describes geometrical ideas like line, line segment, open and closed figures, angle, triangle, quadrilateral, circle, etc., with the help of examples in surroundings.
- demonstrates an understanding of angles by
  - identifying examples of angles in the surroundings.
  - classifying angles according to their measure.
  - estimating the measure of angles using 45°, 90°, and 180° as reference angles.
- demonstrates an understanding of line symmetry by
  - identifying symmetrical 2-Dimensional (2-D) shapes which are symmetrical along one or more lines.
  - creating symmetrical 2-D shapes.
- classifies triangles into different groups/types on the basis of their angles and sides. For example- scalene, isosceles or equilateral on the basis of sides, etc.
- classifies quadrilaterals into different groups/types on the basis of their sides/angles.
- identifies various (3-D) objects like sphere, cube, cuboid, cylinder, cone from the surroundings.
- describes and provides examples of edges, vertices and faces of 3-D objects.
- finds out the perimeter and area of rectangular objects in the surroundings like floor of the class room, surfaces of a chalk box, etc.
- arranges given/collected information such as expenditure on different items in a family in the last six months, in the form of table, pictograph and bar graph and interprets them.

Class VII

The learner

- multiplies/divides two integers.
- interprets the division and multiplication of fractions.
- for example interprets as of . Also is interpreted as how many make?
- uses algorithms to multiply and divide fractions/decimals.

- solves problems related to daily life situations involving rational numbers.
- uses exponential form of numbers to simplify problems involving multiplication and division of large numbers.
- represents daily life situations in the form of a simple equation and solves it.
- adds/subtracts algebraic expressions.
- distinguishes quantities that are in proportion. For example, tells that 15, 45, 40, 120 are in proportion as is the same as
- solves problems related to conversion of percentage to fraction and decimal and vice-versa.
- calculates profit/loss percent and rate percent in simple interest.
- classifies pairs of angles based on their properties as linear, supplementary, complementary, adjacent and vertically opposite and finds value of the one when the other is given.
- verifies the properties of various pairs of angles formed when a transversal cuts two lines.
- finds unknown angle of a triangle when its two angles are known
- explains congruency of triangles .
- on the basis of the information given about them like (SSS, SAS, ASA, RHS).
- using ruler and a pair of compasses constructs, a line parallel to a given line from a point outside it and triangles
- finds out approximate area of closed shapes by using unit square grid/graph sheet.
- calculates areas of the regions enclosed in a rectangle and a square.
- finds various representative values for simple data from her/his daily life contexts like mean, median and mode.
- recognises variability in real life situation such as, variations in the height of students in her class and uncertainty in happening of events like throwing a coin.
- interprets data using bar graph, such as consumption of electricity is more in winters than summer, runs scored by a team in first 10 overs, etc.

Class VIII

The learner

- generalises properties of addition, subtraction, multiplication and division of rational numbers through patterns.

- finds out as many rational numbers as possible between two given rational numbers.
- proves divisibility rules of 2, 3,4, 5, 6, 9 and 11.
- finds squares, cubes and square roots and cube roots of numbers using different methods.
- solves problems with integral exponents.
- solves puzzles and daily life problems using variables.
- multiplies algebraic expressions, for example, expands $(2x-5)\ (3x2+7)$.
- uses various algebraic identities in solving problems of daily life.
- applies the concept of per cent in profit and loss situation in finding discount, VAT and compound interest, for example, calculates discount per cent when marked price and actual discount are given or finds profit per cent when cost price and profit in a transaction are given.
- Solves problems based on direct and inverse proportions
- Solves problems related to angles of a quadrilateral using angle sum property.
- verifies properties of parallelograms and establishes the relationship between them through reasoning.
- represents 3D shapes on a plane surface, such as sheet of paper, blackboard, etc.
- verifies Euler's relation through pattern.
- constructs different quadrilaterals using compasses and straight edge.
- estimates the area of shapes like trapezium and other polygons by using square grid/graph sheet and verifies using formulas.
- finds the area of a polygon.
- finds surface area and volume of cuboidal and cylindrical object.
- draws and interprets bar charts and pie charts.
- makes hypotheses on chances of future events on the basis of its earlier occurrences or available data like, after repeated throws of dice and coins

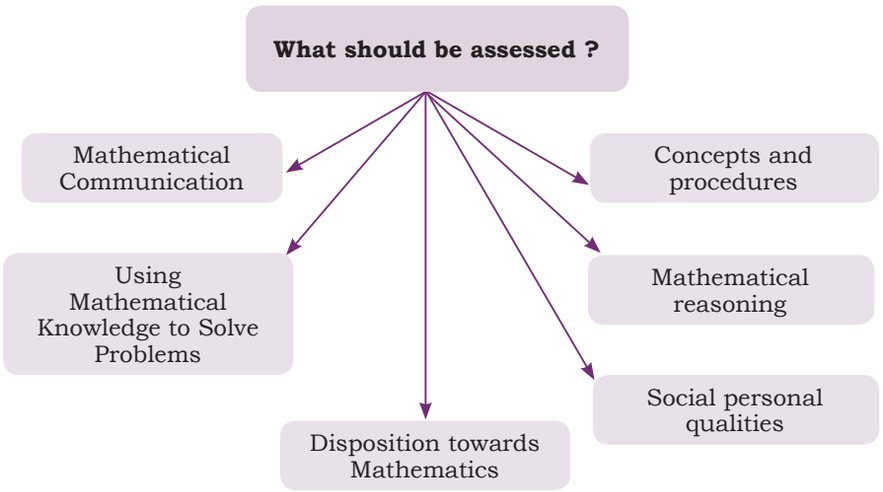
Module 9

Assessment in Mathematics at the Elementary Stage

The focus of assessment of mathematics learning at primary stage should be on how children learn. The assessment would be on

- understanding of how children learn mathematics.
- development of the mathematical concept and their application in daily life.
- development of the social personal qualities of the children.

Let us now reflect on what needs to be assessed broadly in mathematics. These areas are presented in the diagram below



The most common question asked while teaching mathematics at elementary stage is—"What should be assessed with regard to the mathematical learning?"

Ask your self what it is that you are looking for. As far as mathematical learning is concerned, if we look at the broader perspective of school mathematics, it is generally accepted that children must find mathematics as something to "talk about to communicate through, to discuss among themselves, to work together on".

The student must be assessed with regard to the following capabilities

Are they able to—

- consolidate and reason out mathematical facts, figures, etc.?
- use abstraction to perceive relationships, to see structures, to argue logically the truth and falsity of statements?

- understand the basic structure of different branches of mathematics, such as arithmetic, algebra, geometry, data handling, mensuration, etc.
- familarise themselves with different ways of dealing with and handling abstraction and their generalisation?
- apply mathematical concepts learnt at this stage to solve problems related to daily life situations?
- cooperate and help each other while performing a group activity.

The various tools for assessment explained in the module on School Based Assessment (SBA) are to be used judiciously integrating them with day to day teaching-learning processes. Some examples have been given below to illustrate the teaching learning of mathematics at elementary stage to accomplish the desired competencies as laid down in the learning outcomes.

Exemplar I

This activity may be conducted in the last year of pre-school or in the beginning of class 1.

Learning Outcome

Using number names for groups having 1 to 9 objects

Objective of the activity

The child would be able to—

- Tell number names corresponding to a given group
- Create number names from 1 to 10 in order

Material Required

Appropriate material can be made available that supplement the poem/story, Audio/visual aids.

Procedure

- Usually children are exposed to numbers before they enter the classroom. For example, they have heard there elders are probably using number names in their daily life like, 'I need one more biscuit, give me Rs. 5, we will visit Dadi's house after 2 days', etc.
- Some children may already know the number names (random and/or in order). The objective of the activity is to introduce children with the number names or counting names preferably in order using a context to which a child can co-relate.
- Reciting small poems (appropriate for a class 1 learners), songs, games or stories which included numbers names

(random and also given in order) can be done in the classroom to help learners grasp the number names in order.

- One of the poems is given below. Teachers can find many such poems in their local language.

One One One
One little baby
Eating a bun.

(Show objects which exist in 1 like—nose, sun in the sky etc., and ask children to touch or show the body parts and objects that exist in number 1)

Two Two Two
Two little children
Go to the zoo.

(Show objects which exist in group of 2 like—eyes, wheels in a bicycle etc., and ask children to touch or show the body parts and objects that exist in group of 2)

Three Three Three
Three green parrots flying free.
Four Four Four

Four hungry pups
Want to eat more.

Five Five Five
Five honey bees are
Going to the hive

(Continue reciting the poem and associating group of objects up to number 9)

Provide opportunities to all the children in the class including those who hesitate to participate. Children with different abilities should be asked to cite examples according to their observations.

Exemplar II

Primary school students usually experience difficulties in understanding the geometrical concept of angle. Why do they experience the difficulties?. Most of the time this difficulty happens due to the way the teacher helps the students in understanding the concept. Instead of defining an angle using mathematical terminology given in some standard textbooks,

we should make use of different and visible daily life situations, such as—

- Intersections at road junctions, in scissors, hands of a clock, etc.
- Corners of a table top, match box corners, etc.
- Bending of an arm or leg.
- Slopes, ramps, roofs, hillsides, railway signals, slides in children's park; and
- Rotation in opening and closing of a door, a wiper, ceiling fan, spokes of a wheel, etc.

Such situations offer a lot of opportunities to develop the concept of angle in a systematic manner. It is all the more important to study the concept of angle since it is fundamental to various shapes to be learnt in geometry and allied topics in elementary classes. Hence this exemplar suggests few activities to develop the concept and meaning of an angle and classification of angles into acute, right, and obtuse angles and some activities for assessment of the learner progress.

Key Concepts

Angle, measure of an angle, classification of angles, comparison of angles

Learning Outcomes

- Explores the idea of angle
- Classifies angles into right angle, acute angle and obtuse angle

In order to achieve these learning outcomes students may be divided into groups that will comprise of boys and girls almost equal in number. The group can be engaged in the following activities.

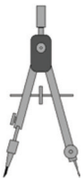
Activity 1

- Ask the students to observe the two hands of a clock at different times and describe the openings between two hands.

- Ensure that the clocks are such that the hands (i.e. needles) of the clock, the numbers on them etc. can be felt by the hands after touching them.
- Ask the students to observe and express in terms of the kind of angle formed between the hands of the clock, shown the figures presented. Can students identify more such objects that involve angle measurements and their relations?
- This activity can also be performed by arranging the students in a circle assigning different numbers from 1 to 12. They may hold a card with the number embossed on it for recongnising the number and help her/him to estimate the angle.
- The centre can be decided
- The hand of the clock can be formed by making two rows of students originating from the centre and going in different directions towards the boundary of the circle to form an angle between them.
- The child at the centre and at the boundary will keep on telling about their position and the card held by them.
- The angle can be changed by rearranging one or both the rows.
- Student with disabilities can become a part of the boundary or the hand.

Activity 2

Ask the learners to observe objects like scissors, compasses, divider, etc., and to describe the openings in terms of angle formation in their own words.



Activity 3

Ask the learners to observe the opening and closing of a door in the room and to describe it in terms of angles made by movement of the door at different points during its motion in their own words. Children should be provided opportunities to touch the objects and make ideas about the angle made at the points.

Activity 4

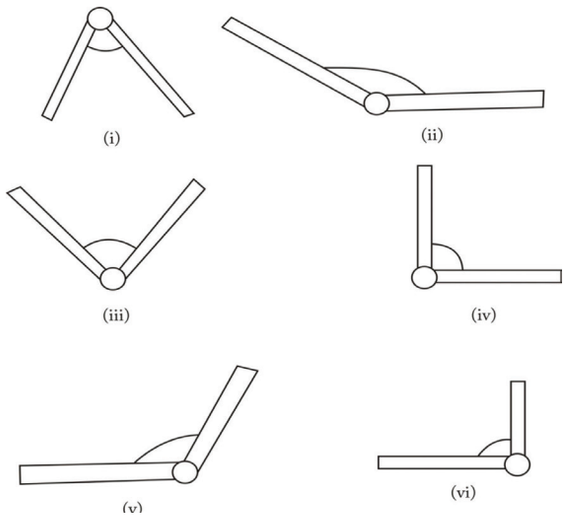
Ask the learners to observe the opening and closing of a lid of a box and to describe the angles visible in their own words.

Teacher may use such activities, to establish the concept of an angle as a figure formed by two rays with a common initial point; the two rays are called arms of the angle, and the common initial point its vertex.

It may be emphasised that, while naming an angle, vertex is always in the middle; and arms moving around.

Activity 5

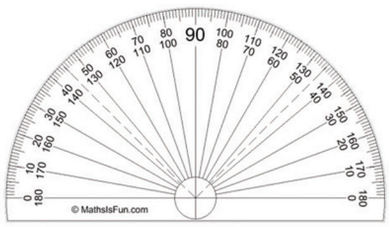
- Ask the learners to take some wooden/plastic/broom sticks and a few uniformly cut pieces of rubber tubes
- They should be asked to join pairs of sticks using pieces of rubber tubes as shown below; to explain the measures of angles. Instead of this students may use a pair of scissors or divider.
- Encourage the learners to notice the openings in the above pairs of sticks (Take any two at a time).
- They will be able to notice that in the cases of (i) and (ii) or (ii) and (iii) or (iii) and (iv) or (iv) and (v) or (v) and (vi), it is easy to decide as to "which opening is greater" or "which opening is smaller". That is, in geometrical terms, to decide which angle is greater or which smaller.

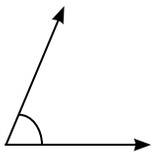


- Let them observe that the opening of pairs of sticks in cases (i) and (iii) or (ii) and (v) or (iv) and (vi) it is difficult to decide which opening is greater or which opening is smaller. In other words, which angle is greater or which smaller.
- Such activities may arouse among learners a need for measuring a variety of angles.
- This activity can be played by visually challenged students also.

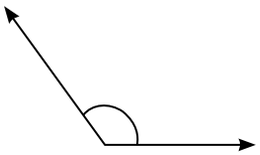
Activity 6

- Take out a protractor from geometry box and show it to the learners. The lines and numbers on the protractor can be embossed so that children read them by moving their fingers on them. Let all the learners feel that a general protractor is semicircular through to touch.

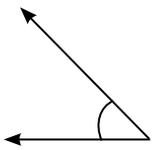




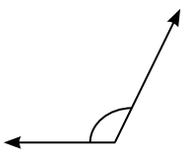
(i)



(ii)



(iii)



(iv)

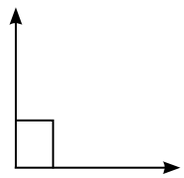
- Ask the learners to observe two types of scales on the edge of protractor (0°–180° and 180°–0°)
- Inform that angle is measured in "degrees" (symbolically denoted as °). Also, demonstrate how a protractor is used to measure angles.
- Learners may be asked to measure the following types of angles or to draw some angles and measure them. Children may also be provided angles formed by joining plastic/wooden sticks and to measure the opening.
- Help the learners measure these types of angles and make them realise the need and use of two types of scales on the edge of the protractor.

Activity 7

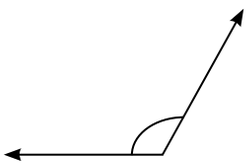
- Encourage the students to observe the angles formed at each corner of their books, notebooks, room, etc. and decide whether the angle can be measured in each of the cases using a protector. Allow them to discuss and choose more such objects from their surroundings.
- Let them measure the angle at each corner using a protractor and note it down.
- Introduce the term "right angle" for an angle having a measure 90°.

Activity 8

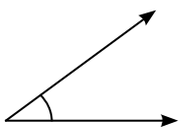
- Students may be divided into four or five mixed groups of boys and girls.
- Set of angles given below may be given to each group (May be drawn on a paper or cut outs)



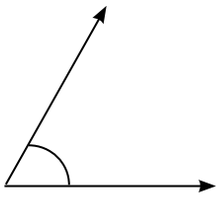
(i)



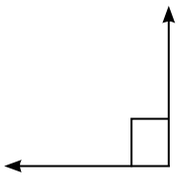
(ii)



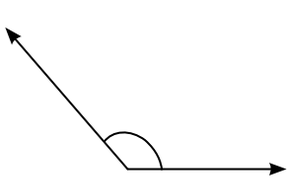
(iii)



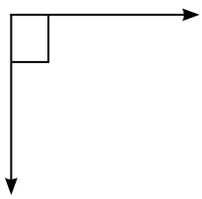
(iv)



(v)



(vi)



(vii)

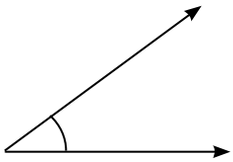
- Encourage each group to measure the angles using protractor and note down in a chart paper.
- Let them observe the measurements and see that angles in figures (i), (vii) and (v) are right angles.
- Let them also observe (It could be done through a hand made protractor using a paper also as shown in Class VI NCERT mathematics textbook) that angles (iii) and (iv) are less than (smaller than) a right angle and more then 0°; and angles (ii) and (vi) are greater than (larger than) a right angle and less then 180°.
- Explain that angles (iii) and (iv) are termed as acute angles and angles (ii) and (vi) as obtuse angles.
- Learners may also perform activities given in Mathematics Kit and Laboratory Manual in mathematics for class I to VIII, (prepared by NCERT, New Delhi).

There are specially designed rulers protractors and compasses for visually challenged students to do the above activities.

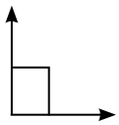
Assessment — Misconceptions and Support mechanism

Comparing Angles

- Students may compare a pair of the following type of angles—

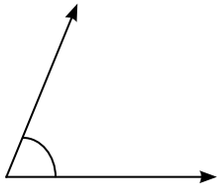


A

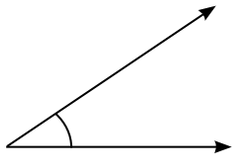


B

- Some learners may say that angle A is greater than angle B, because arms of angle A are longer than that of B. If it is so, it may he pointed out that angles are not **compared according to lengths of arms**. They are compared according to the **openings between their two arms**.
- Similarly, ask the learners to compare a pair of angles of the following type—



P



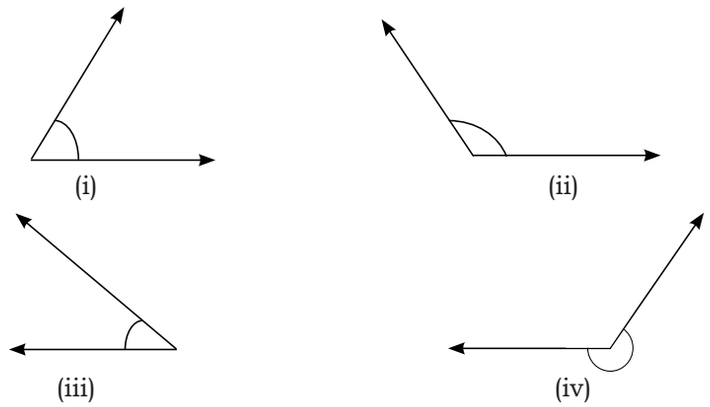
Q

- Some learners may say that angle Q is greater than angle P, because arc of angle Q is greater than that of angle P. Here,

again, it may be pointed out that angles are not compared according to lengths of arcs. They are compared according to the openings between their two arms.

Measuring an Angle

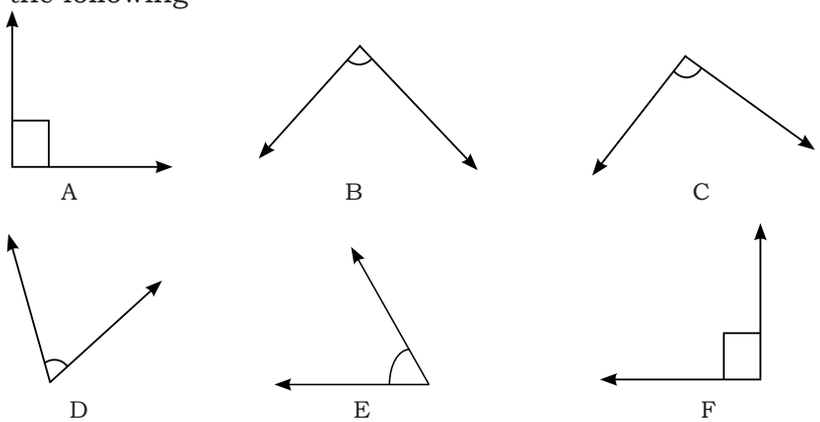
- Ask them to measure the following types of angles.



- Some of the learners may measure angle (iii) as 150° instead of 30°. Similarly, some learners may measure angle (iv) as 60° instead of 120°. If it is so, point out to the learners to use proper scale on the edge of the protractor.
- Some learners may measure the above angle by placing the straight edge of the protractor on one of the arms of the angle instead of placing (0–180° or 180°–0) line on the arm. If so, ask learners to place the protractor in a proper way.

Right Angle

- Ask the learners to identify right angles from among the following—



- Some of the learners may identify only angle A as right angle. However in this case; angles A, F and C are right angles and

others are not right angles. It means that learners are not recognising angle F or C as a right angle. Reason for this may be that learners are often presented with prototypical examples of right angles. As a result, they do not recognise other representations of the angle. Therefore, to remove this deficiency, teacher must present right angles and also other angles in every type of orientations.

The Upper Primary Stage

Exemplar 3

Learning outcomes

- In this exemplar we would like to discuss the concept of division of fractions with a whole number as well as with another fraction.

Division of a whole number with a fraction

Let us see how we can find a solution in the following situation—There are 2 bars of chocolates. Each bar is broken in ½ parts. One piece is to be given to on students. How many students will get a piece?

After enough discussion students should reach the conclusion that we need to find both the bars in half pieces and so 2 ÷ ½ is to be found.

They may further find that the number of such pieces is 4.

That is 2 ÷ ½ = 4. That is, 4 students will get such a piece each.

What will happen if it is ⅓ part of each bar? Similar discussion will lead to 2 ÷ ⅓ = 6

On the same lines 2÷ ¼ = 8.

These can be written as 2 ÷ ½ = 4; 2 ÷ ⅓ = 6; 2 ÷ ¼ = 8 and so on. Students may be asked if they observe any pattern in this. If so, they may be asked 3 ÷ ¼ = ?; 2 (½) ÷ ⅓ = ?

Students may be motivated to observe the pattern and find the algorithm now.

Word problem—In a school there are 5 sections in class VII. For going to a field trip the class teachers were asked to divide their respective section into 5/6th parts. If each part requires one bus, how many buses are required for the field trip?

Solution—The number of buses required for the field trip is equal to the total number of the parts in 5 sections. That is given by

$$5 \div \frac{5}{6} = 5 \times \frac{6}{5} = 6$$

Therefore 6 buses are required for the field trip.

Exemplar 4

Learning Outcome

The learner multiplies two integers

The learner is aware about the integers, their addition and subtraction. The concept of multiplication is to be introduced now. For doing this student may be engaged in doing certain activities that will make the students understand the process of multiplication on of integers.

Children are sufficiently aware about patterns. We can think of activities that are based on patterns and the concepts that are already known to the students. This will engage them in evolving the concept of product of two integers.

Let us start with introducing the product of the type say, positive integer x negative integer = negative integer

Activity 1

Take the case 2 x (–3)

The children have learnt multiplication of natural numbers in their primary classes. The concept of addition of numbers was used then. For example 2 × 3 will mean adding 3 twice, i.e., 3 + 3 = 6. Teacher may ask the students to recall this process of addition. some more examples may be discussed such as 5 × 4, 6 × 8 etc.

It is introduced only after teaching students about addition of numbers.

Students may now be asked to discuss if the same process can be used to find 2 x (–3). They are aware about addition of integers. After sufficient discussion the class may settle down to

So, 2 x (-3) = (–3) + (–3) = –6

During the discussion it may be observed whether the students are able to link the idea of multiplication of numbers studied in primary classes. The discussion may focus on how to do this.

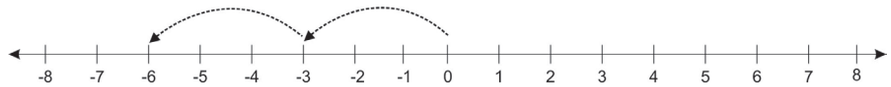
Similar examples may be given to give them an idea of the product of a positive and a negative integer.

Another way of doing this is as suggested in the following activity.

Activity 2

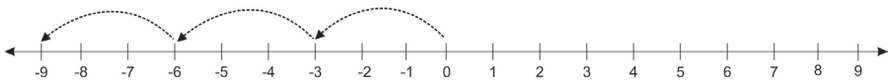
The same idea of adding –3 twice can also be shown on a number line.

–3 x 2



Two jumps of 3 each in the negative direction means (–3) + (–3) and they take us to –6.

–3 × 3



(–3) + (–3) + (–3) will mean three jumps towards negative direction. It can be seen that we reach -9.

This visual representation of addition of integers will attract the students attention and they will get engaged in not only finding the products but also interpret the sign of the resulting product.

The students may be motivated to draw their inference.

Take examples of different such products.

Students may be involved in another activity so that they can get a different way of getting the same concept.

Activity 3

Patterns can be used to arrive at the same result.

For getting –3 × 2 we begin with ,say, 3 × 2.

It is known to the learners that 3 × 2 = 6 ------ (1)

2 × 2= 4 ------- (2)

1 × 2 = 2 ------- (3)

0 × 2 = 0 ---------(4)

A discussion may be held to see if some pattern evolves in getting the products 6, 4,2,0. It may be settled down to the fact that ,in step (2), 4 = 6 – 2; in step (3), 2 = 4 – 2 ; in step (4) , 0 = 2 – 2 It implies that at every successive step 2 is being subtracted from the product. So carrying this logic further we get,

–1 × 2 = 0 – 2 = –2

–2 × 2 = (–2) – 2 = –4

–3 × 2 = (–4) – 2 = –6

So finally we get , (–3) × 2 =-6

More number of such products may be discussed using this method.

Now we take the case of (negative integer) x (negative integer).

It may be discussed whether the technique used in earlier activities can be used here also. The process of patterns may finally be thought of.

Let us take the product (–3) × (–2).

Activity 4

Let us start with (–3) × 3 = –9 ... (1)

This is now known to students done in earlier activities.

(–3) × 2 = –6 ...(2)
(–3) × 1 = –3 ...(3)
(–3) × 0 = 0 ...(4)

Ask the students to observe the pattern in the products obtained. Here you can assess whether the students have understood about the product of a negative integer and a positive integer done just prior to this process and if they are able to apply it here or not. Moreover the teacher may also see whether the students are able to see the pattern in the products and how they extend it further.

It may be seen that, in step (2), –6 = (–9) + 3; in step (3), –3 = (–6) +3; in step (4), 0 = (–3) + 3 .

Having observed this pattern we extend it further as

(–3) × (–1) = 0 + 3 = 3
(–3) × (–2) = 3 + 3 = 6

So finally we get (–3 × (–2) = 6.

Some more examples may be given to the students to work out. Students should also be encouraged to create examples and give it to each other.

Students may be asked about the type of integer (i.e., whether positive or negative) they get in both the above types of products discussed .The observation made by the students may be assessed. This may further be supplemented by asking students about the following—

What is the sign of the integer obtained for the products,
(–1) × (–1) ; (–1) × (–1) × (–1) ; (–1) × (–1) × (–1) × (–1) ; (–1) × (–1) × (–1) × (–1) × (–1) ;
(–1) × (–1) × (–1) × (–1) ×(–1) × (–1), (–2) × (–3) (–5) .....

Having introduced the product of two integers it is essential that students get used to such a kind of multiplication. Instead of giving merely exercises to multiply two integers they may be given a game to play that requires multiplying integers many times.

Game

Games can give a sense of suspense, joy, frustration and fun. Using games in teaching mathematics leads to an improved attitude towards mathematics, enhanced motivation and support for the development of children's problem solving abilities The mathematical discussions that happen while playing mathematical games leads to the development of mathematical understanding.

Refer to Game1 on page 12 of Class VII mathematics textbook.

On a large sheet of paper small boxes are drawn (similar to the game of snakes and ladders or chess) with 19 rows and 11 columns. Numbers from –104 to 104 are written in them.

- Take a bag containing two blue and two red dice. The number of dots on one of the coloured dice may represent positive integers and on the other coloured dice negative integers. The students may be allowed to decide about that. In the present instance suppose number of dots on red coloured dice represent positive integers and the other colour negative integers.
- Divide the class into different groups. The group size may decided with the help of students. Students in each group will play the game. Suppose the group size is say, 4.
- Each player will place his/her counter on the 0 mark.
- Each player will shuffle the dice in the bag and without looking at it take out two dice .The dice may be of the same colour or different colours.
- The player has to multiply the numbers on the dice taken out.
- If the dice are of the same colour the product of numbers would be positive, otherwise it will be negative.
- If the product is positive the player should move the counter towards 104 and if it is negative the counter should be moved towards –104
- The one who reaches either 104 or –104 is the winner.

While playing this game the students would be multiplying the integers ,several times and watch others in their group doing so. The students will also become familiar with the facts related to the multiplication of integers.

As a teacher you get an opportunity to know whether the students are able to work with product of integers.

Exemplar 5

Learning Outcomes

finds surface area of cuboidal objects.

finds volume of cuboidal objects.

Learning Objectives

After completion of this activity the students will be able to-

Form cube and cuboid and obtain formula for their surface area.

Obtain a formula for finding the volume of a cuboid.

Activity 1

Materials Required

Cardboard, ruler, cutter, cello tape, sketch pen/pencil, white paper, chart paper.

Procedure

Students may be divided into groups of four in each group.

Students should be given cardboard, ruler, cutter, cello tape, sketch pen/pencil, white paper, chart paper etc.

Students should be encouraged to make a shape involving six identical squares each of side a unit. (Fig. 1)

Fold the squares along the lines markings to form a solid as shown in (Fig. 2)

Discussion/Demonstration

It may be discussed what kind of solid has been obtained in Fig. 2. It is a cube. The students may tell why it is a cube. This will give an opportunity to the teacher to see whether students know about the attributes of a cube.

Each face of the cube is a square of side 'a'. Therefore, the area of one face of the cube is a^2.

Total number of surface in a cube is 6.

Therefore, surface area of a cube is $6a^2$.

Activity 2

Discussion

How many sides are there in a cuboidal object?

Students, after observing and discussing about different such shapes like,eraser , box, room, etc., in their surroundings may talk about a cuboid and answer the above query. It will also make the teacher see how far students know about a cuboid.

Observe the following cuboid having sides a, b and c—

Students have learnt about areas of 2D shapes. They may be asked to tell what is the shape of each face? Are all the faces

of the same shape? Which of these are same which are not same.

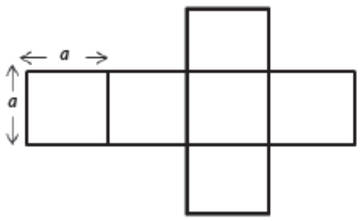


Fig.1

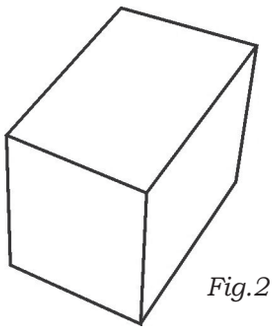


Fig.2

Then they may talk about the present 3D shape with the given dimensions.

Enough time and discussion may be held to see about the term surface area of a solid. Students should realise that it is the sum of areas of all the faces.

Area of Front/back face of cuboid having sides a & b unit = ab

Total area of front and back faces = $ab+ab=2ab$

Total area of both the side faces of a & c unit = ac+ac=2ac

Total area of top and bottom faces of b & c unit = bc+bc=2bc

Total surface area= 2ab+2bc+2ca = 2(ab+bc+ca)

If a=b=c then total surface area= $2a^2+2a^2+2a^2= 6a^2$

Activity 3

Materials required

Net of a cuboid, Plastic/clay, cutter, ruler, cardboard.

Procedure

Take a net of a cuboid of length l, breadth b and height h (say l=4, b=4, h=3).

- Fold it to form an open cuboid.
- Fill this cuboid with clay/ plasticise and Remove the net.
- Place the cuboid in the cardboard and cut it into four equal pieces along its length l (Fig. 4)

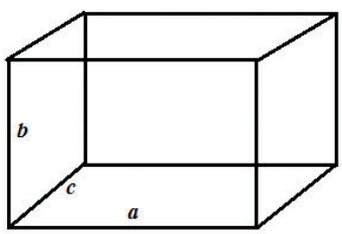


Fig.3

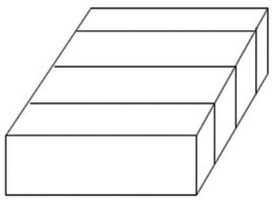


Fig.4

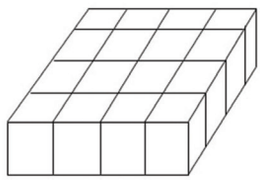


Fig.5

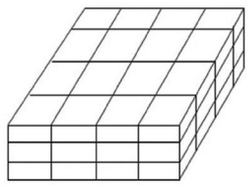


Fig.6

- Cut the cuboid with equal pieces along its breadth b (Fig. 5)

Cut the cuboid into three equal pieces along its height h (Fig. 6)

Discussion

The cuboid is divided into cubes of unit length.

The number of unit cubes so formed is 48, which can be expressed as $4 \times 4 \times 3$.

Students may tell about the volume of a cube with dimensions $1 \times 1 \times 1$.

Volume of cuboid is = $4 \times 4 \times 3$ cubic units i.e., $l \times b \times h$.

Similarly, students may be asked to form cuboids of different dimensions say, 2,4,4 units; 3, 4, 5 units etc. and asked to observe the number of cubes.

This activity can be used in explaining formula for the volume of cuboid which is $l \times b \times h$. During the discussion teachers may observe how students approach the generalised formula.